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THE FUTURE OF SECONDARY INSTRUCTION IN GEOMETRY

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If the future of secondary instruction in geometry is to be judged in the light of the past, two thousand years may elapse without any material change. Horace boasted of a monument more enduring than bronze. *Euclid's Elements* has stood like a granite monolith against the intellectual erosion of centuries. This book has seen the decline of great civilizations, the rise and fall of empires, and the birth of great religions. It has passed unscathed through countless successions of languages and dialects. Such is its perfection of detail that all attempts at improvement during its two milleniums of existence have led only to a return to its original form and substance.

And yet the very existence of so perfect a creation of human thought has barred the path of progress in secondary teaching in geometry during the past century. So complete a picture does Euclid present that his teaching has come to be regarded as the embodiment of absolute truth. Only more recent investigations into the nature of symbolic logic have shown that there do exist certain serious lapses from logical accuracy in the Euclidean text, which have been, as a rule, aggravated in many of its more recent imitations. This is an awful blow to philosophical rigorists, but will be welcomed by true lovers of mathematics. A human Euclid is more to be appreciated!

A second result of these investigations will, it is hoped, take the form of a conviction that it is utterly hopeless to attempt to impress the beginner with the refinements of the pure logic of space, or with those of Euclid. The teacher of geometry, to appreciate this fact, must make studies of this character for himself. No teacher should flatter himself that he is competent to instruct pupils in elementary geometry until he has at least made a survey of the field of pure geometry. The reaction of such a study upon elementary instruction will depend largely upon circumstances. Geometry, more than any subject taught in secondary schools, is "hard boiled." The vicious circle of the college entrance examination, the textbook written to fit the same, the teacher trained to teach from the book, and the normal course designed to enable the teacher to do this, is almost perfect. What changes, whether for better or worse, may be noted in geometry teaching during the past half-century?

One change stands out in the memory of many now living (and teaching) and as well from an examination of a chronological sequence of published texts. It is an increased emphasis upon "originals."

Let it be noted at once that most "originals" are not original at all, as far as the student is concerned. Not a little difficulty in regard to college entrance examinations in geometry lies in the enormous difference, from the examiner's point of view, between a theorem which the candidate has seen before and one which he has not seen. Secondary school teachers realize this difference most deeply, and pay tribute to its significance by combing over innumerable texts for geometric puzzles to present to their classes in the hope of covering all the ground which is likely to afford material for examiners.

It is no disrespect to Euclid or his remarkable book to say that time thus spent is for the most part wasted. The writer is not competent to enter upon a critical discussion of the question of transfer of mental training from one field of thought to another. But the secondary curriculum is overcrowded. If a great many matters of common knowledge are to be covered properly in high school, matters which were unknown to earlier generations, then there is no time to waste upon false intellectual notions. Secondary mathematics in America has in the past eschewed Cartesian analysis, all mention of the conic curves, all reference to the principles of the calculus, and any attention to the real significance of complex numbers. These things were not "required for college." Instead we have had so-called "originals," such as the following:

"If AB and AC are the equal sides of an isosceles triangle ABC, and if on BA the distance BE is taken from B, and on AC produced the distance CD is taken equal to BE, then the line DE is bisected by the base BC."

The stupendous importance of this fact has doubtless escaped the reader's attention, as it has the writer's. If secondary geometry training actually reached the point where this exercise could be demonstrated on a moment's notice, perhaps something could be said for its kind. But usually this is not the case. Instead, the exercise constitutes merely an additional theorem, and its solution is based upon a device not readily thought of.

Exercises such as these have really prolonged the course in plane geometry and have altered the status of the subject from that of a classic to that of a bag of tricks.

If the secondary course in mathematics is to be a real introduction to this branch of learning, the time devoted to this sort of thing had better be spent upon something else. This means that the course in geometry admits of material reduction in time and extent. If the student is given a fair introduction to geometric concepts in the junior high school course, as is most commendably urged by the National Committee, it comes within the bounds of possibility that the course in plane and solid geometry be reduced to one year in time.

This procedure would involve material changes in the character of the subject. In the first place, the distinction between plane and solid geometry, while essential to Euclid's development, is immaterial from a pedagogic viewpoint. Again, certain important geometric relations, such as line-symmetry, which are now all but entirely omitted, open the way for elimination of many of the tedious circumlocutions of the present order of procedure.

Hilbert showed plainly enough the necessity of making Euclid I:4 over into an axiom (or assumption). Not to enter into the details of this question, it would appear equally logical and practical to assume that any two lines, or angles, or polygons, or curved figures, or solid figures which possess line-symmetry are congruent. From this it easily follows that every case of congruence is reducible to symmetric relations, and a large number of geometric theorems follow almost directly.

Symmetry and parallelism are really forms of correspondence. The last word in pure geometry makes the subject a science of correspondence of points. Similarity is another form of correspondence. Similarity in two and in three dimensions is identical. If geometry teachers could be persuaded to strain at a few less gnats and swallow a few more camels, the plane and solid goemetry could be combined into one subject, for which one year's time would be ample. Most of the familiar theorems of solid geometry are no more difficult than much of the plane geometry, and they are no more numerous than the "originals" which now have such a vogue.

Another economy would result from the omission from the plane geometry of a whole chapter of theorems relating to "numerical properties of triangles." These theorems are interesting, but, except possibly for the two which measure the third side of an oblique triangle, they are not essential.

Another section which could be omitted without being missed is that relating to the golden section and the inscription of the regular decagon in the circle. This problem is of interest only in so far as it relates to the question of the superior limits of elementary mathematics in the solution of the pure equation $x^n - 1 = 0$.

In the solid geometry, time would be saved by agreement to accept some form of Cavalieri's theorem as a fundamental assumption.

There is also a question as to whether the course in solid geometry might not be shortened by detaching those theorems which relate to polyhedral angles and spherical polygons, including triangles. Spherical geometry really signifies very little unless it is studied in connection with spherical trigonometry, and applied to astronomy and navigation, a chapter which belongs distinctly to college curricula. Spherical geometry as a pure science, that is, as a special case of elliptic geometry, is not an appropriate subject for secondary teaching, and the subject is not so treated in American texts.

But it should be emphasized that the greatest economy of time would result from a radical departure from the customary formalism of the subject as now presented in most American high schools. To cite an instance of this, it is not sufficient to say that two vertically opposite angles are equal because they are supplements of the same angle adjacent to both. One must needs go through the ritual of drawing two intersecting lines, lettering the angles by means of five letters, and solemnly repeating steps and ''authorities'' in a column, or worse, in two columns. This is supposed to be in the interest of perfection of logic. Consider Euclid's first proposition, for example. It may not be known to some readers that this is a problem, viz.: ''To construct an equilateral triangle on a straight line.'' He is very careful to state precisely how the two circles are to be drawn, to distinguish between the lines *AB* and *BA* as radii, and to give authorities for every step of the proof that his triangle is equilateral. But he wholly omits to prove that his circles meet at all, nor does he give any ''axiom'' or ''postulate'' covering this case!

A change of spirit from the excessive formalism of geomtery to something more humane and in keeping with other subjects in the secondary curriculum will be brought about, if at all, through the medium of new textbooks. It goes without saying that it will meet with violent opposition from many who feel that plane geometry ought to be a sheet-anchor in the stormy sea of loose argument which prevails in politics and too often even in natural science. But the fact is that at present the mode of presentation of the subject gives it an unearthly aspect, a thing so wholly apart from the world of real existence that good people boast of their unfamiliarity with it.

But once the idea is fully grasped that the chief aim of geometry teaching is to afford an analytical insight into the world of sense, the secondary curriculum will afford no better subject than a year of plane and solid geometry.